Multiplicative-noise-induced coherence resonance via two different mechanisms in bistable neural models

Jun Tang,[1,2] Ya Jia,[1,*] Ming Yi,[1] Jun Ma,[1] and Jiarong Li[1]
[1]*Department of Physics and Institute of Biophysics, Huazhong Normal University, Wuhan 430079, China*
[2]*College of Science, China University of Mining and Technology, Xuzhou 221008, China*
(Received 24 October 2007; revised manuscript received 19 January 2008; published 6 June 2008)

The bistable FitzHugh-Nagumo (FHN) neural model driven by two multiplicative noises and one additive noise is investigated. Two different potential mechanisms for enhancing coherence of bistable FHN model are presented, that is, the first multiplicative noise changes the system from the bistable to the oscillatory regime, and the second multiplicative noise can enhance the symmetry of two stable states of the system. The two mechanisms are analytically or numerically explained. At any level of the second multiplicative noise, a maximal coherence have been found at some intermediate noise intensity of the first multiplicative noise. Only when the first multiplicative noise intensity is less than 0.0001 can a maximal coherence be obtained at some intermediate noise intensity of the second multiplicative noise. These coherence resonance phenomena have been understood in terms of the presented mechanisms.

DOI: 10.1103/PhysRevE.77.061905 PACS number(s): 87.10.−e, 05.40.Ca, 05.70.Fh

I. INTRODUCTION

Rhythm generation is a very important problem in nonlinear science, particularly in biological systems. The most familiar rhythm generation is offered by limit cycles [1]. In addition, frequently transiting between two steady states can also be seen as a kind of rhythmic behavior [2]. Even when the system is monostable, internal rhythm can be generated by the effect of noise, which drives the system to an oscillatory [3] or a bistable regime [4,5]. The response of the nonlinear system without an external periodic signal may be enhanced through an optimal amount of noise. This phenomenon is called coherence resonance (CR) [3]. CR is originally found in a simple dynamical system in the vicinity of a saddle-node bifurcation [6] and is due to nonuniformity of the noise-induced limit cycle [7]. CR has been extensively studied in many nonlinear systems [3,8–10].

Much biological signaling is realized through rhythmic behaviors of the systems [11,12], so enhancing the coherence of biological systems is a crucial problem in bioinformatics. For example, some optimal characters of biological signaling systems, such as the number of molecules [13], the number of ion channels in clusters [12], and the volume of cells [14], are theoretically calculated under the condition of optimal coherence, and these optimal results often agree qualitatively with experimental observations. Therefore, understanding how the biological systems enhance their coherence is very important for one to comprehend their biological characters. The coherence of the FitzHugh-Nagumo (FHN) neural model [15] has been intensively studied [16]. Increasing experimental evidence has established that certain types of neurons frequently operate in a bistable regime [17]. It has been shown that the CR phenomenon can be observed both in the symmetrical bistable FHN neural model [2] and in the unsymmetric case [18]. The mechanisms of coherence enhancement for both cases are utterly different: one is the breaking of symmetry [19] induced by noise, and the other is based on the restoration of symmetry induced by multiplicative noise [18]. Furthermore, the FHN model has various dynamical behaviors such as monostable, bistable, and oscillatory, referring to different biological contexts [3,18,20]. Now a question to be raised is as follows: can noise shift the FHN model from the bistable to the oscillatory regime, and is the coherence enhanced when this noise-induced transition is accomplished?

In this paper, based on a stochastic modified FHN model driven by two multiplicative noises and one additive noise, we report here that one multiplicative noise can change the system from the bistable to the oscillatory regime; coherence of the system is greatly enhanced by the change and a maximal coherence is observed at some intermediate intensity of this multiplicative noise (i.e., CR phenomenon). The other multiplicative noise can enhance the coherence via noise-induced restoration of symmetry, and corresponding CR is also observed. Based on these two coherence-enhancement mechanisms, the interaction of two multiplicative noises is studied. Our results show that each of the two multiplicative noises affects CR induced by the other one. This paper is organized as follows. In Sec. II, a stochastic version of the modified FHN model is introduced. In Sec. III, we demonstrate the multiplicative noise-induced transition from the bistable to the oscillatory regime; then the CR phenomenon induced by this noise is explained by an approximate approach in the framework of a small-noise expansion. In Sec. IV, the other multiplicative noise-induced restoration of symmetry and corresponding CR phenomenon is studied; then the mechanism is qualitatively explained. The interaction of two multiplicative noises is studied in Sec. V. We end with conclusions in Sec. VI.

II. MODEL

We consider the following version of the FHN model:

$$\varepsilon\frac{du}{dt}=u(u-1)(a-u)-bv, \tag{1}$$

*Corresponding author. jiay@phy.ccnu.edu.cn

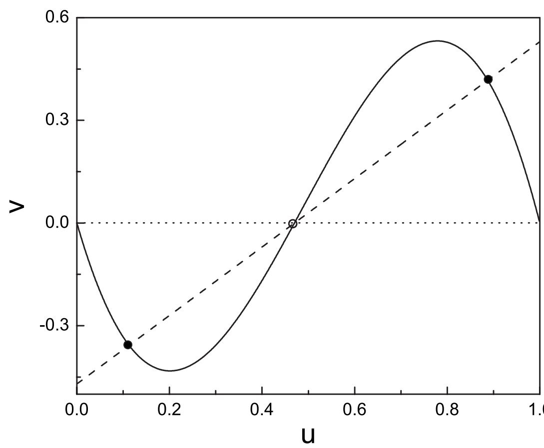

FIG. 1. Nullcline plot of the FHN model for typical parameter values: $a=c=0.47$, $z=1.0$, $b=0.1$. Dashed line: v nullcline ($\dot{v}=0$), solid line: u nullcline ($\dot{u}=0$). Solid circle: stable fixed point, empty circle: unstable fixed point.

$$\frac{dv}{dt}=z(u-c)-v, \quad (2)$$

where $u(t)$ represents the membrane potential of the neuron and $v(t)$ is related to the time-dependent conductance of the potassium channels in the membrane [21]. ε determines a different time scale for u and v, and u is much faster than v if $\varepsilon \ll 1$. a, b, z, and c are constant parameters that determine the dynamics exhibited by the system, which can vary between monostable, oscillatory, and bistable. In this paper, we use the parameters $\varepsilon=0.01$, $a=c=0.47$, $b=0.1$, and $z=1.0$, for which the deterministic system has two unsymmetric stable fixed points (see Fig. 1).

In reality, neurons are permanently affected by different kinds of noise sources. In many previous works about biological systems [9], noises originate in the random variation of one or more of the control parameters, such as the rate constants associated with a given set of reactions. We now assume the two parameters a and z are subjected to additive random fluctuations, i.e., $a \to a+\xi(t), z \to z+\eta(t)$. The two additive fluctuations give rise to two multiplicative noise terms in the FHN equations, and the different effects of two multiplicative noises will be discussed in Secs. III and IV, respectively. The assumption of such noise sources is not unreasonable. The FHN equations constitute a qualitative simplification of the well-known Hodgkin-Huxley (HH) model of electrical signaling in neurons. In the FHN model, every parameter is related to one or more externally controllable parameters in the HH model. Our simplified choice of the multiplicative noise terms corresponds to parameter fluctuations originating from external control on the neuron. Furthermore, as in most studies of coherence resonance, the additive noise is inserted in the slow-variable equation. The stochastic version of FHN model Eqs. (1) and (2) is given by

$$\frac{du}{dt}=\frac{1}{\varepsilon}[u(u-1)(a-u)-bv]+f(u), \quad (3)$$

$$\frac{dv}{dt}=z(u-c)-v+u\eta(t)+\zeta(t), \quad (4)$$

where the multiplicative noises are interpreted in the Stratonovich sense, $f(u)=\frac{1}{\varepsilon}u(u-1)\xi(t)$, and $\xi(t),\eta(t),\zeta(t)$ are mutually uncorrelated Gaussian white noises. The statistical properties are given by

$$\langle \xi(t)\rangle=0, \quad \langle \xi(t)\xi(t')\rangle=2\alpha\delta(t-t'), \quad (5)$$

$$\langle \eta(t)\rangle=0, \quad \langle \eta(t)\eta(t')\rangle=2\beta\delta(t-t'), \quad (6)$$

$$\langle \zeta(t)\rangle=0, \quad \langle \zeta(t)\zeta(t')\rangle=2\gamma\delta(t-t'), \quad (7)$$

where α, β, and γ are the corresponding noise intensities. We use a fourth-order Runge-Kutta integration scheme with a time step 0.001, and the numerical algorithm presented by Sancho *et al.* [22] will be used to simulate the noises.

III. NOISE-INDUCED CHANGE FROM THE BISTABLE TO THE OSCILLATORY REGIME

Our goal is to study the effects of two multiplicative noises, so intensity of the additive noise $\zeta(t)$ is fixed at $\gamma=3\times10^{-3}$ throughout this paper. In this section, we focus on the multiplicative noise term $f(u)$, so intensity of another multiplicative noise $\eta(t)$ is fixed at $\beta=10^{-4}$.

Time evolutions of v for different intensity α of the multiplicative noise $\xi(t)$ are plotted in Fig. 2(a). For very low α [$\alpha=10^{-6}$ in the top of Fig. 2(a)], the system possesses bistable behavior, and the additive noise $\zeta(t)$ induces jumps between the two stable states. When α is increased to an intermediate value [$\alpha=10^{-3}$ in the center of Fig. 2(a)], the system is frequently changing between the oscillatory and the bistable regime. Sometimes the system jumps between two stable states, sometimes it stays in the oscillatory regime. When α is large enough [$\alpha=3.5\times10^{-3}$ in the bottom of Fig. 2(a)], no jumps between stable states are observed, and the system always stays in the oscillatory regime. These results demonstrate that a strong enough multiplicative noise term $f(u)$ in Eq. (3) can induce a change from the bistable to the oscillatory regime.

To analytically explain this change from the bistable to the oscillatory regime, an approximate approach is used. In this approach, the systematic contribution of $f(u)$ can be incorporated explicitly into Eq. (3) as the first-order term of a small-noise expansion [18,19]. The effective equation for Eq. (3) can be written in the same form as Eq. (1),

$$\varepsilon'\frac{du}{dt}=u(u-1)(a'-u)-b'v, \quad (8)$$

where values of the renormalized parameters ε',a',b' will be derived in Appendix A. It is obvious that, although the noise is directly added to parameter a, the fluctuation affects all parameters. It must be pointed out that the above analytical results are only valid for very weak noise, otherwise the renormalized model parameters will be negative (for example, when $\alpha \geq 5\times10^{-3}$).

Then three values of noise intensity α corresponding to Fig. 2(a) are selected, and the nullclines of the effective sys-

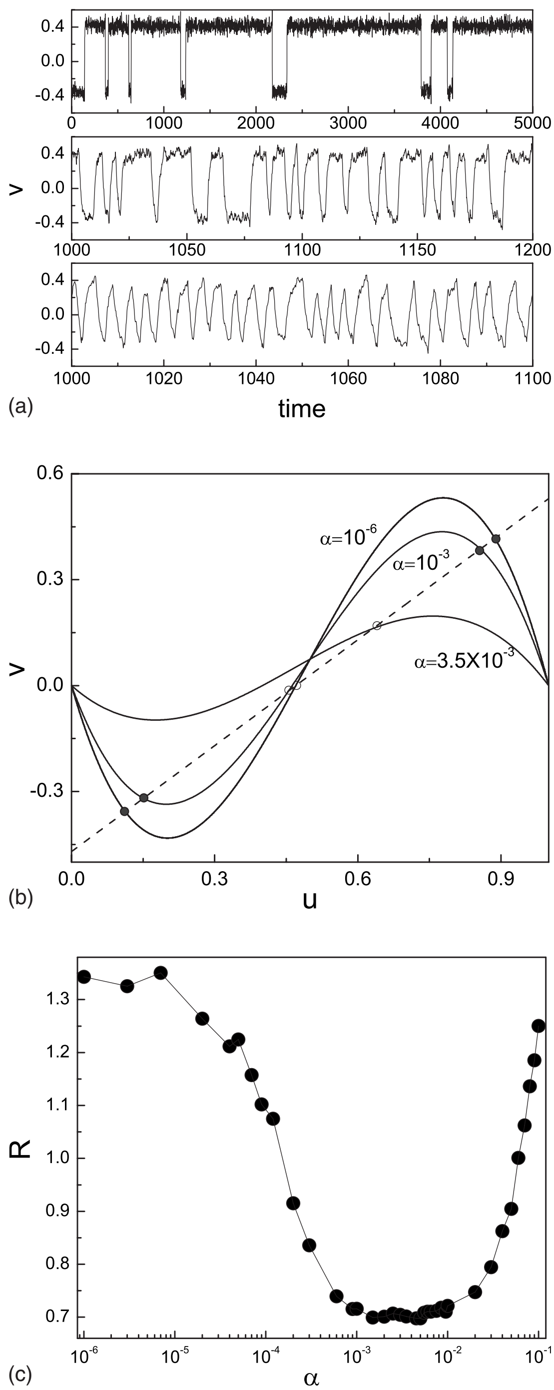

FIG. 2. (a) Top to bottom: time series of v for $\alpha=10^{-6}, 10^{-3}, 3.5\times10^{-3}$. (b) Nullclines of the effective FHN model. Dashed line: v nullcline; three solid lines: u nullclines, respectively, for $\alpha=10^{-6}$, 10^{-3}, and 3.5×10^{-3}. (c) Coherence parameter R vs multiplicative noise intensity α.

tem involving Eqs. (8) and (2) for these values are plotted in Fig. 2(b). For $\alpha=10^{-6}$, the u nullcline intersects with the v nullcline three times (two stable and one unstable fixed point), i.e., the system possesses bistable behavior. For $\alpha=3.5\times10^{-3}$, both nullclines intersect only once (one unstable fixed points), i.e., the system possesses oscillatory behavior. The critical value of α for the system changing from the bistable to the oscillatory regime is about 2.5×10^{-3}. For $\alpha=10^{-3}$, although the system still possesses bistable behavior, the additive noise can casually drive it to the oscillatory regime because $\alpha=10^{-3}$ is close to the critical value.

To quantitatively measure the coherence enhancement, the normalized variance of periods T_i is calculated. The periods T_i are intervals between two adjacent transitions from maximum v to minimum v. The normalized variance, which is called the coherence parameter, is defined as [13]

$$R=\frac{\sqrt{\langle (T_i-\langle T_i\rangle)^2\rangle}}{\langle T_i\rangle}. \tag{9}$$

The dependence of R on the multiplicative noise intensity α is shown in Fig. 2(c). It is clearly seen that R first decreases to some minimum value and then increases again. This is the coherence-resonance phenomenon, and the minimal R corresponds to the highest degree of coherence in the system. With α gradually increasing to the critical value 2.5×10^{-3}, the system is easier to drive to the oscillatory regime, and it is more coherent. In a region close to the critical value of α, coherence of the system is always in a high level, and the minimum R or the highest degree of coherence corresponds to the most regular oscillation. However, larger α will destroy the regularity of oscillation, which leads to the final increase of R.

IV. NOISE-INDUCED ENHANCEMENT OF SYMMETRY OF TWO STABLE STATES

In this section, we will study the effect of another multiplicative noise term $u\eta(t)$, so the intensity of multiplicative noise $\xi(t)$ is fixed at $\alpha=10^{-6}$.

In the work of Zaikin *et al.* [18], an elaborate multiplicative noise term is used to change the symmetry of two stable states. An intermediate amount of multiplicative noise optimizes the symmetry of two stable states, and jumps of the system between two states are perfectly equidistant. As a result, the additive noise is most effective in producing coherence, since the potential barrier heights (and thus the corresponding escape times) are the same in the two jump directions. Here, we will show that the multiplicative noise term $u\eta(t)$ originating from random parameter fluctuation can bring about the same results. To exhibit the effect of $u\eta(t)$ in changing symmetry, parameters a and c are set equally at 0.47, for which the two stable states of the system (respectively with lower and higher v) are unsymmetric.

The additive noise $\zeta(t)$ can induce jumps between the two stable states. In Fig. 3(a), time series of v are shown for three selected values of noise intensity β. For $\beta=5\times10^{-4}$, the system spends roughly all the time in the upper stable state. The threshold for a transition from the upper stable state to the lower stable state is larger than the threshold for the inverse transition. In this case, the two stable states of the system are unsymmetric. For $\beta=4\times10^{-3}$, the system spends almost the same time in both stable states. So the thresholds for transitions between the two stable states are approximately of the same size, i.e., the two stable states of the system are symmetric. For $\beta=5\times10^{-2}$, the threshold for a transition from the upper stable state to the lower stable fixed

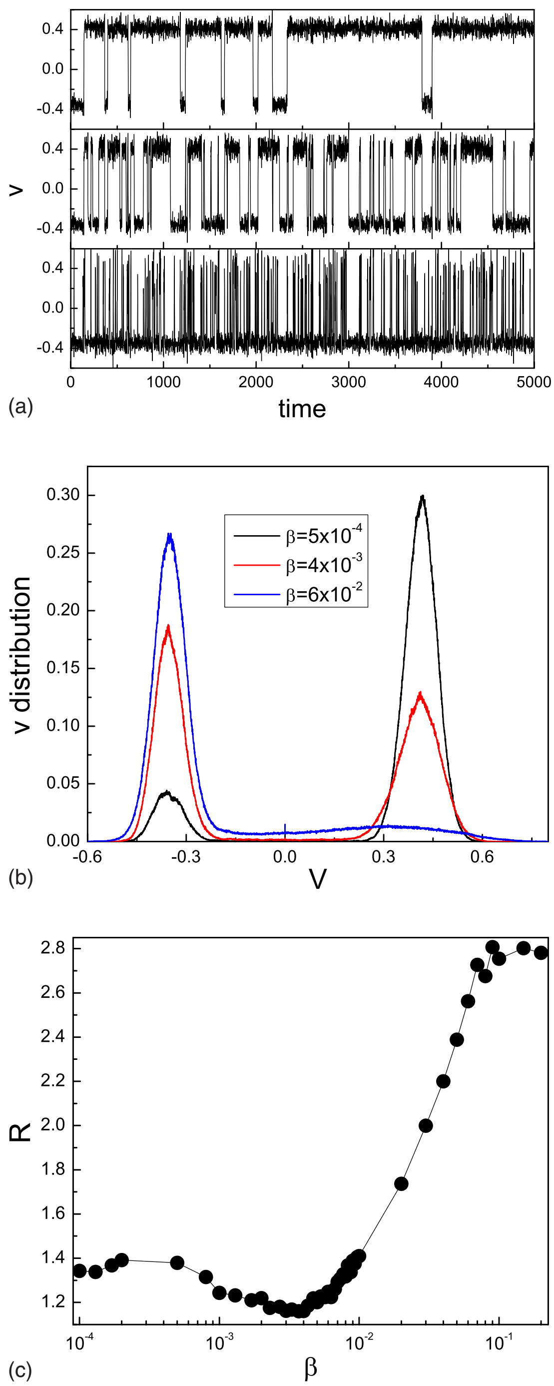

FIG. 3. (Color online) (a) Top to bottom: time series of v for $\beta = 5\times10^{-4}$, 4×10^{-3}, 6×10^{-2}. (b) v distribution for three different β corresponding to (a). (c) Coherence parameter R vs multiplicative noise intensity β.

point is smaller than the threshold for the inverse transition; the system prefers to stay in the lower state. Now the two stable states of the system become unsymmetric again. To exhibit more explicitly the noise-induced symmetry changing, the distributions of v over a period of time are plotted for three cases in Fig. 3(b). For $\beta=4\times10^{-3}$, the system spends almost the same time in two states that are symmetric. For $\beta<4\times10^{-3}$, the symmetry of the two stable states is gradually restored by increasing β. On the contrary, when

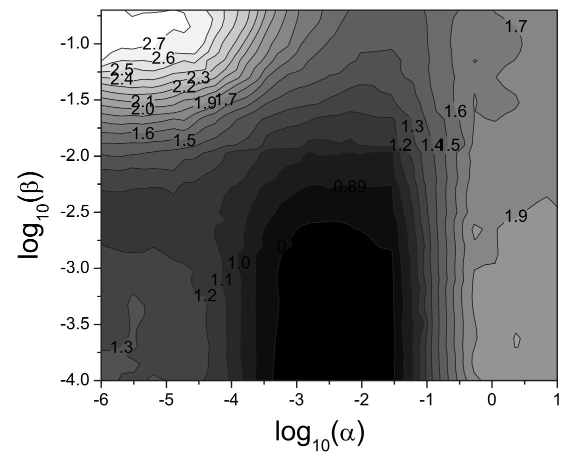

FIG. 4. Coherence parameter R as a function of combination $(\log_{10}(\alpha), \log_{10}(\beta))$.

$\beta>4\times10^{-3}$, the symmetry of the two steady states is destroyed by β. In fact, in the extreme case when β is large enough, the upper stable state becomes unstable and the system becomes monostable or excitable.

The coherence parameter R is plotted against noise intensity β in Fig. 3(c). It is clearly seen that R first decreases to some minimum value and then increases again. Obviously, this is a coherence resonance phenomenon. The minimum R corresponds to the noise-induced symmetry of the stable states, in which case the additive noise is most effective in producing coherence, since the potential barrier heights are the same in the two jump directions. Comparing Fig. 2(c) with Fig. 3(c), it can be seen that the optimal value of R in Fig. 2(c) is less than that in Fig. 3(c), i.e., the coherence of oscillation is much better than that of transition between two stable states.

V. COMBINED EFFECT OF TWO MULTIPLICATIVE NOISES ON COHERENCE RESONANCE

It has been shown that each multiplicative noise can induce coherence resonance, and the mechanisms are discussed, respectively, in the above sections. In what follows, we will focus on how the two multiplicative noises, or two different mechanisms, affect each other.

Simulations have been performed for different α and β, and the corresponding coherence parameter R's are calculated. In Fig. 4, contour of R is plotted in the plane $(\log_{10}(\alpha), \log_{10}(\beta))$, in which darker gray corresponds to smaller values of R.

First, when β is fixed at an arbitrary value, the coherence parameter R exhibits a minimum (R_m) for some intermediate value of α. Three typical values, 10^{-4}, 3×10^{-3}, and 10^{-1}, of fixed β are chosen, and corresponding R's are plotted against α in Fig. 5. It is clearly seen that larger fixed β leads to larger values of R, and a prominent minimum of R can be observed for one fixed β. For arbitrary fixed β value, R decreases to the minimal value (R_m) when the intensity of multiplicative noise term $f(u)$ is large enough to change the system from the bistable to the oscillatory regime. In Fig. 6,

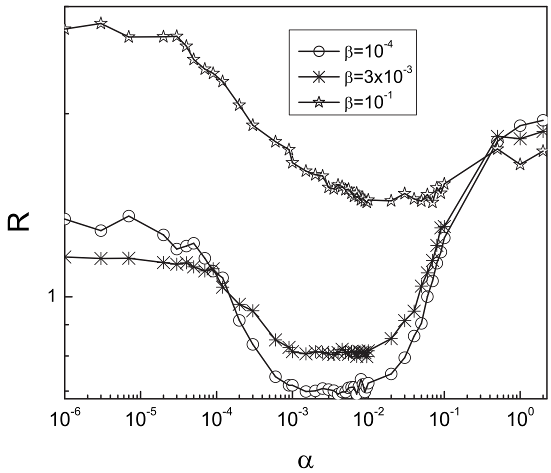

FIG. 5. Coherence parameter R vs multiplicative noise intensity α for three typical values of β.

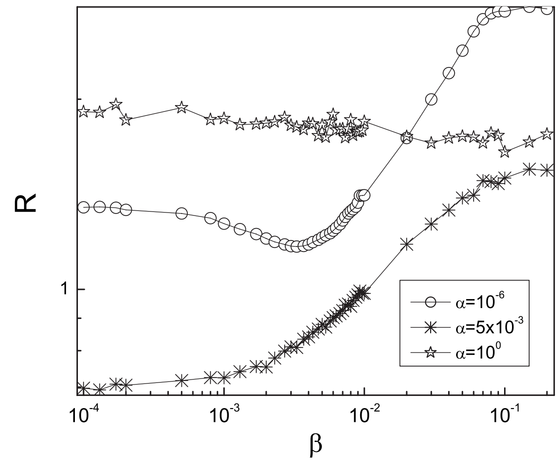

FIG. 7. Coherence parameter R vs multiplicative noise intensity β for three typical values of α.

R_m monotonically increases with β because the regularity of oscillation is destroyed by the noise term $u\eta(t)$. In addition, although the two multiplicative noises can interact with each other, there is no perfect double coherence resonance as shown in Refs. [18,24].

Secondly, only when α is fixed at a region of $\alpha < 1 \times 10^{-4}$ (see the left part of Fig. 4) does the coherence parameter R exhibit a minimum for some intermediate values of β. For $1\times 10^{-4} < \alpha < 3\times 10^{-1}$, R will monotonically increase with β, i.e., no CR is observed; for $\alpha > 3\times 10^{-1}$, R is very large for arbitrary β. In Fig. 7, three typical values, $10^{-6}, 5\times 10^{-3}$, and 10^{0}, of fixed α are chosen, and corresponding coherence parameter R is plotted against β. Only $\alpha = 10^{-6}$ is in the region where CR can be observed.

For very small α, $f(u)$ is not able to change the system to the oscillatory regime, and the system is in the bistable regime. An intermediate intensity of $\eta(t)$ can make two stable states totally symmetric, which corresponds to the minimal value of R. For larger α ($> 1\times 10^{-4}$), $f(u)$ is able to change the system to (or close to) the oscillatory regime, i.e., the system is not transiting between two stable states. Certainly, the above mechanism for the noise term $u\eta(t)$ to induce CR is unreasonable. Moreover, because the noise term $u\eta(t)$ can destroy the regularity of oscillation, R monotonically increases with β. When α is too large ($\alpha > 3\times 10^{-1}$), the regularity of oscillation is destroyed by $f(u)$ itself, so R is very large and almost constant when β is increased.

In Fig. 8, R_m is plotted against noise intensity again. R_m exhibits a minimum for intermediate values of α, which is the character of double coherence resonance [24]. It must be pointed out that not all R_m is obtained for intermediate values of β, because in the region of $\alpha > 1\times 10^{-4}$, no CR is found.

VI. CONCLUSION

In summary, two potential mechanisms for multiplicative noises to enhance coherence are discussed in the bistable FHN model. First, one multiplicative noise term $f(u)$ can change the system from bistable to oscillatory regime. Due to this coherence-enhancement mechanism, a coherence resonance phenomenon is induced by the noise term $f(u)$, and the optimal coherence corresponds to the most regular oscillation. This mechanism can be analytically understood in the framework of a small-noise expansion. Secondly, mo-

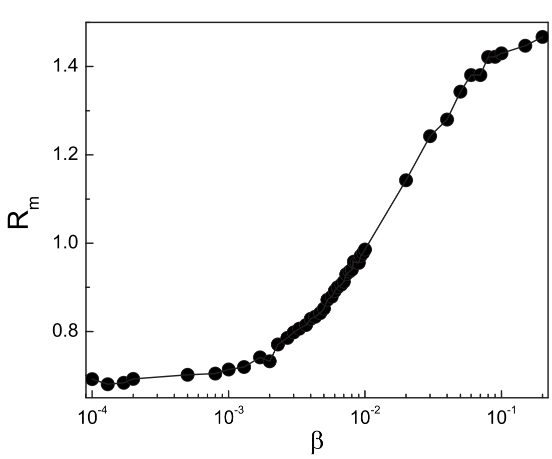

FIG. 6. Minimal values of coherence parameter R_m, obtained for fixed β's, versus β.

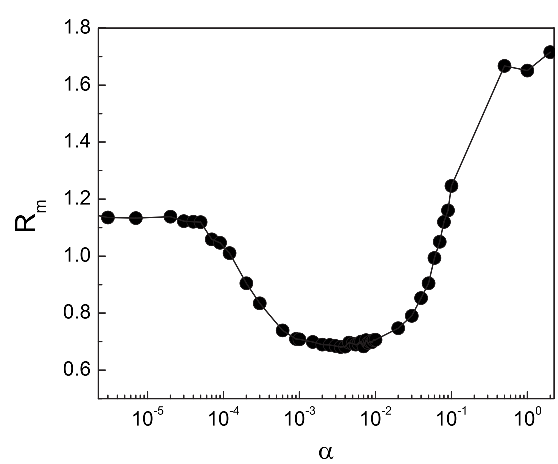

FIG. 8. Minimal values of coherence parameter R_m, obtained for fixed α's, versus α.

tivated by the pioneering work of Zaikin *et al.* [18], the other multiplicative noise term $u\eta(t)$ is added to change the symmetry of two stable states of the bistable system. For an intermediate value of multiplicative noise intensity β, $u\eta(t)$ can drive the two stable states to be totally symmetric, which corresponds to optimal coherence, i.e., another coherence resonance is induced by the multiplicative noise term $u\eta(t)$. Finally, the interaction of the two multiplicative noises, or two different mechanisms, is investigated. The results show that each of the two multiplicative noises can affect coherence resonance induced by another one. And in some region of noise intensities, the system exhibits the character of double coherence resonance. All the results can be easily understood in terms of the two coherence-enhancement mechanisms.

Our study has been performed based on a general version of the FHN model in the bistable regime, which is realistic for biological systems, and hence we believe that the presented mechanisms are operating in certain kinds of real neurons, even in other biological systems. It must be pointed out that the additive noise $\zeta(t)$ has not been intensively studied in this paper. In the work of Zaikin *et al.* [18], double coherence resonance [24] is induced by a combination of multiplicative and additive noises. We predict that each of the two multiplicative noises can induce double coherence resonance together with the additive noise, and this is a motivation of our future work.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant Nos. 10575041 (Y.J.), 10675052 (J.R.L.), and 10747005 (J.M.).

APPENDIX A: DERIVATION OF THE EFFECTIVE PARAMETERS

In Sec. III, to explain the noise-induced changing from the bistable to the oscillatory regime, an approximate approach was introduced. In order to establish the effect of the multiplicative noise term $f(u)$, we note that $f(u)$ has a nonzero mean equal to [19,23]

$$\langle f(u)\rangle = \frac{\alpha}{\varepsilon^2}\langle u(u-1)(2u-1)\rangle, \tag{A1}$$

where the angular brackets denote averaging over the probability distribution of the multiplicative noise. According to Eq. (A1), the random term $f(u)$ gives rise to a systematic nonzero contribution to the average dynamics of the system. This systematic contribution can be incorporated explicitly into Eq. (3) as the first-order term of a small-noise expansion [18,19], where the remaining stochastic contributions of the noise average out to zero. The effective equation for Eq. (3) can be written as

$$\frac{du}{dt} = \frac{1}{\varepsilon}[u(u-1)(a-u) - bv] + \frac{\alpha}{\varepsilon^2}u(u-1)(2u-1). \tag{A2}$$

To be compared with Eq. (8), Eq. (A2) is transformed to

$$\frac{du}{dt} = \frac{\varepsilon - 2\alpha}{\varepsilon^2}\left[u(u-1)\left(\frac{a\varepsilon - \alpha}{\varepsilon - 2\alpha} - u\right) - \frac{\varepsilon b}{\varepsilon - 2\alpha}v\right]. \tag{A3}$$

Then, comparing Eq. (A3) to Eq. (8), we can obtain the effective parameters

$$a' = \frac{a\varepsilon - \alpha}{\varepsilon - 2\alpha}, \tag{A4}$$

$$b' = \frac{\varepsilon b}{\varepsilon - 2\alpha}, \tag{A5}$$

$$\varepsilon' = \frac{\varepsilon^2}{\varepsilon - 2\alpha}. \tag{A6}$$

[1] A. T. Winfree, *The Geometry of Biological Time* (Springer, Berlin, 1980).

[2] B. Lindner and L. Schimansky-Geier, Phys. Rev. E **61**, 6103 (2000).

[3] A. S. Pikovsky and J. Kurths, Phys. Rev. Lett. **78**, 775 (1997).

[4] M. Samoilov, S. Plyasunov, and A. P. Arkin, Proc. Natl. Acad. Sci. U.S.A. **102**, 2310 (2005).

[5] C. Stambaugh and H. B. Chan, Phys. Rev. B **73**, 172302 (2006).

[6] G. Hu, T. Ditzinger, C. Z. Ning, and H. Haken, Phys. Rev. Lett. **71**, 807 (1993).

[7] W. J. Rappel and S. H. Strogatz, Phys. Rev. E **50**, 3249 (1994).

[8] S. G. Lee, A. Neiman, and S. Kim, Phys. Rev. E **57**, 3292 (1998); G. Giacomelli, M. Giudici, S. Balle, and J. R. Tredicce, Phys. Rev. Lett. **84**, 3298 (2000); C. Zhou, J. Kurths, and B. Hu, *ibid.* **87**, 098101 (2001); C. Palenzuela, R. Toral, C. R. Mirasso, O. Calvo, and J. D. Gunton, Europhys. Lett. **56**, 347 (2001).

[9] M. Yi and Y. Jia, Phys. Rev. E **72**, 012902 (2005); M. Yi, Y. Jia, Q. Liu, J. R. Li, and C. N. Zhu, *ibid.* **73**, 041923 (2006).

[10] Z. H. Hou and H. W. Xin, Phys. Rev. E **60**, 6329 (1999); Z. H. Hou, L. F. Yang, and H. W. Xin, J. Chem. Phys. **111**, 1592 (1999); Z. W. Wang, Z. H. Hou, and H. W. Xin, Chem. Phys. Lett. **362**, 51 (2002).

[11] Y. Li and J. Rinzel, J. Theor. Biol. **166**, 461 (1994).

[12] J. W. Shuai and P. Jung, Phys. Rev. Lett. **88**, 068102 (2002).

[13] Y. Morishita, T. J. Kobayashi, and K. Aihara, Biophys. J. **91**, 2072 (2006).

[14] Z. H. Hou, J. Q. Zhang, and H. W. Xin, Phys. Rev. E **74**, 031901 (2006).

[15] R. FitzHugh, Biophys. J. **1**, 445 (1961).

[16] B. Lindner and L. Schimansky-Geier, Phys. Rev. E **60**, 7270 (1999); Y. Shinohara, T. Kanamaru, H. Suzuki, T. Horita, and K. Aihara, *ibid.* **65**, 051906 (2002); R. C. Hilborn and R. J. Erwin, Phys. Lett. A **322**, 19 (2004).

[17] S. W. Hughes, D. W. Cope, T. I. Tóth, S. R. Williams, and V.

Crunelli, J. Physiol. (London) **517**, 805 (1999).
[18] A. Zaikin, J. García-Ojalvo, R. Báscones, E. Ullner, and J. Kurths, Phys. Rev. Lett. **90**, 030601 (2003).
[19] J. García-Ojalvo and J. M. Sancho, *Noise in Spatially Extended Systems* (Springer, New York, 1999).
[20] A. Gomez-Marin, J. Garcia-Ojalvo, and J. M. Sancho, Phys. Rev. Lett. **98**, 168303 (2007).
[21] J. Keener and J. Snyder, *Mathematical Physiology* (Springer, New York, 1998).
[22] J. M. Sancho, M. San Miguel, S. L. Katz, and J. D. Gunton, Phys. Rev. A **26**, 1589 (1982).
[23] J. Garcia-Ojalvo, F. Sagues, J. M. Sancho, and L. Schimansky-Geier, Phys. Rev. E **65**, 011105 (2001).
[24] A. A. Zaikin, J. Kurths, and L. Schimansky-Geier, Phys. Rev. Lett. **85**, 227 (2000); Thomas Kreuz, Stefano Luccioli, and Alessandro Torcini, *ibid.* **97**, 238101 (2006).